THE SIMULATION OF PRODUCTION-FINANCIAL PROCESS VIA DIFFERENTIAL EQUATIONS

Abstract: *The article presents one of the possible methods of simulation in business, where the system of differential equations of first order is used. The production-financial system is searched and MATLAB program is used.*

Keywords: *Simulation, production-financial system, differential equations, calculation, MATLAB*

1. Introduction

The simulation plays an important role in searching of production problems. The simulation is the process of attempting to build up the model of the studied system and use it for the purposes of the system behaviour. The production and financial systems pertain to the most complicated dynamical systems. The mathematical models can be used for these purposes. The models of systems are simplified abstract tools which are used for the model systems behaviour. The build-up process is called the modelling and its use for the model systems behaviour is called simulation.

2. Used Methods

For the purposes of simulation the system can be defined as a part of the environment that can be separated by means of thought border. The aim is to build up the simplest model representing the behaviour of the actual model with satisfactory accuracy. The mathematical model can be presented by the system of two differential equation in the form

$$P' = k_{11}*P + k_{12}*C + b_1,$$

$$C' = k_{21}*P + k_{22}*C + b_2,$$

where P means product represented by pieces [p], P' means productivity represented by pieces per time [p/t], C means capital represented by euro [EUR] and C' means capital flow represented by euro per time [EUR/t]. See Tab. 1. The constants are k_{11}, k_{12}, k_{21}, k_{22}, b_1 and b_2.

Production	Finance
Product P [p]	Capital C [EUR]
Productivity P' [pT^{-1}]	Capital flow C' [$EURT^{-1}$]

Tab. 1 System analogy

3. Case study

The case represents the model of production workshop, where the production and financial systems is considered. The constants are $k_{11} = 0$, $k_{12} = 1$, $k_{21} = 0$, $k_2 = 0$, $b_1 = 1$, $b_2 = 2$ in the first case and $k_{11} = 1$, $k_{12} = -1$, $k_{21} = 2$, $k_2 = 1$, $b_1 = 0$, $b_2 = 0$ in the second case. For the purposes of simulation it is marked $P = y(1)$, $P' = y'(1)$, $C = y(2)$, $C' = y'(2)$, thus we can rewrite the described system of differential equations in the form

$$y'(1) = k_{11*}y(1) + k_{12}* y(2) + b_1,$$

$$y'(2) = k_{21}* y(1) + k_{22}*y(2) + b_2.$$

The system of two differential equations is put in program *twoode.m* as a function. See Prog.1a for the first simulation and Prog.1b for the second simulation. The edge conditions are put to *twobc.m* as a function. See Prog.2. It is evident that the initial value, the number of pieces at the beginning of production is 0 and the number of pieces after finishing the process is 48 pieces. The program for calculation and drawing of results *sde.m* is represented in Prog. 3.

```
function dydx = twoode(x,y)
dydx =[0*(1)+1*y(2)+1; 0*y(1)+0*y(2)+2];
```
Prog. 1a Function *twoode.m*

```
function dydx = twoode(x,y)
dydx = [1*y(1)-1*y(2)+0; 2*y(1)+1*y(2)+0];
```
Prog. 1b Function *twoode.m*

```
function res = twobc(ya,yb)
res = [ ya(1); yb(1)-48];
```
Prog. 2 Function *twobc.m*

```
clear all
solinit = bvpinit(linspace(0,8,9),[1 0]);
sol = bvp4c(@twoode,@twobc,solinit);
x = linspace(0,8);
y1 = deval(sol,x);
subplot(2,2,1);
plot(x,y1(1,:));xlabel('Time');ylabel('Production');title('P-F System');
subplot(2,2,2);
plot(x,y1(2,:));xlabel('Time');ylabel('Capital');title('P-F System');
yd11=diff(y1(1,:));
yd12=diff(y1(2,:));
for i=1:99
   xx(i)=x(i);
end
subplot(2,2,3);
plot(xx,yd11(1,:)); xlabel('Time');ylabel('Productivity');title('P-F System');
subplot(2,2,4);
plot(xx,yd12(1,:));xlabel('Time');ylabel('Capital flow');title('P-F System');
```
Prog. 3 Program *sde.m*

It was used the order *bvp4c* of program environment of MATLAB for the solution of the systems of differential equations. The order *bvpint* sets up the initial values of solver *bvp4c*, where the initial grid is [0 8 9], and estimation $ya(1) = 0$, $yb(1) = 48$. The order *bvp4c* makes the calculation. Their input parameters are the function that defines the system of differential equations *twoode*, initial conditions *twobc* and initial estimation *bvpinit*. The evaluation of problem solved by differential equation is stored in variable *y1* after the transfer by order *deval* from the structure stored in variable *sol*, returned by solver. Other orders *plot* and *subplot* serves for drawing the graphs. The order *diff* calculates the first derivations. The results of calculations, the curves of production, productivity, capital and capital flow are presented in graph. The first simulation presents the situation when the production is linearly increasing and thus the number of pieces is increasing. The capital is linearly increasing and cash flow is constant. See Fig.1.
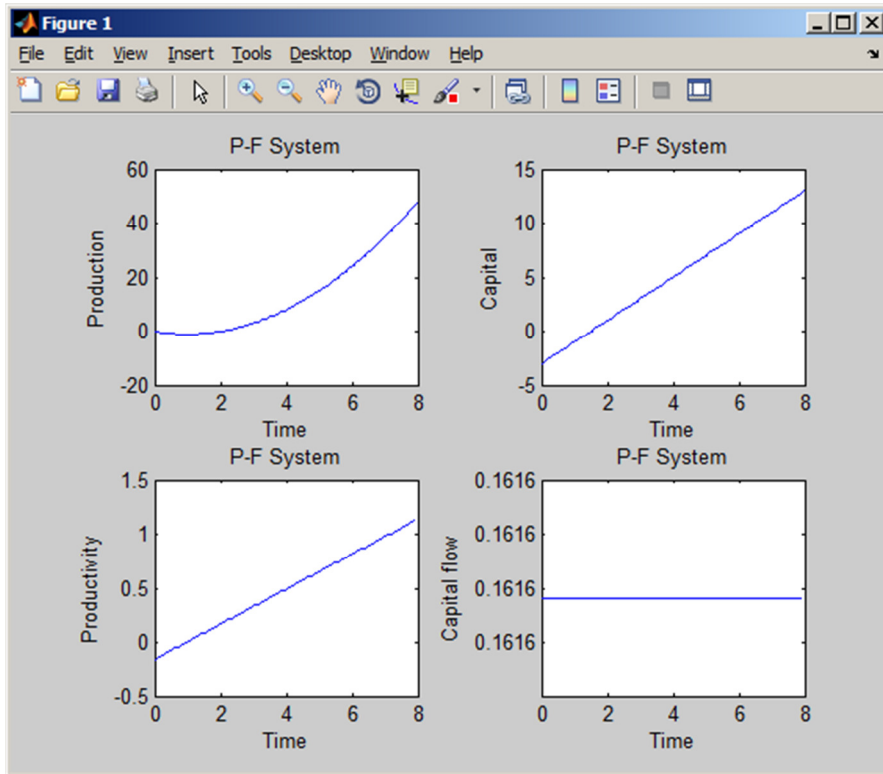
Fig. 1 The curves of product, capital and their derivatives – first simulation

The second simulation presents the situation when the disorder of production is simulated when the scraps are produced during the production. It corresponds to the behaviour of curves of product, capital and their derivatives. The production is not steady and thus the curve of capital and their derivatives.
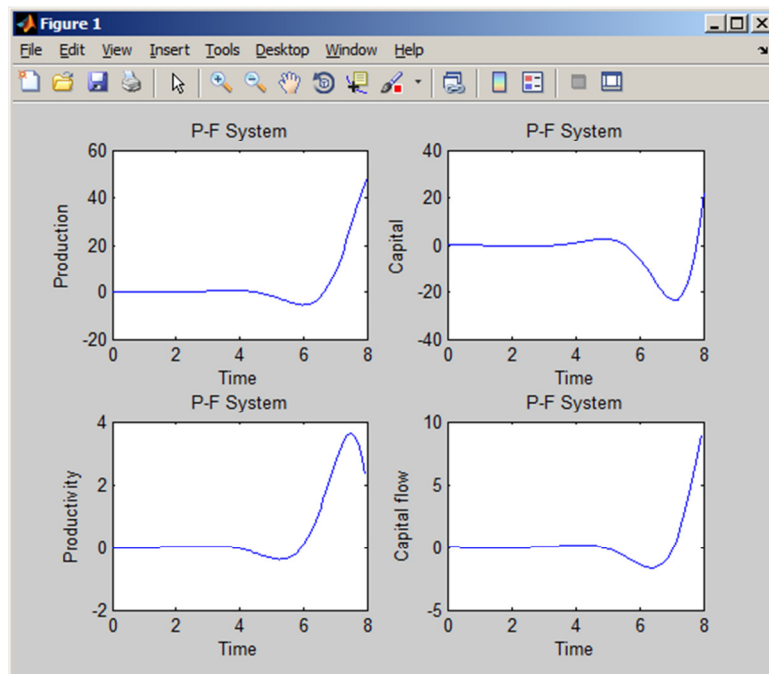

Fig. 2 The curves of product, capital and their derivatives - second simulation

4. Conclusion
The article describes the possible methods of simulation in production, when the system of differential equations of first order is used. It is presented simulation of regular production and with disorder in production when scraps started to be produced. The advantage of this method is that the theory for simulation is very well known. The disadvantage of the methods is indiscreet behaviour of searched values.